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DISTURBANCE-ACCOMMODATING CONTROL THEORY FOR DISCRETE-TIME DYNA--ETC(U)
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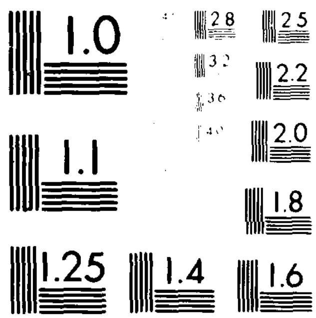

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NATIONAL BUREAU OF STANDARDS

DISTURBANCE-ACCOMMODATING CONTROL THEORY for DISCRETE-TIME DYNAMICAL SYSTEMS.

C. D. Johnson

Final Report,

Contract No. DAAK40-79-M-0028

July, 1979

Dynamic Systems

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SUMMARY

This report describes the extension of Disturbance-Accommodating Control Theory to include dynamical systems described by discrete-time models. Such models are a natural consequence of introducing data-sampling and digital data-processing in conventional analog-type control problems. The theory developed in this report is an essential first step in creating a general purpose control engineering design tool for applying disturbance-accommodating control to digitally-controlled missiles.

Contract Scope of Work

— DISCRETE-TIME DAC THEORY —

C.D. Johnson

General

Extend Disturbance-Accommodating Control Theory, as currently described in the continuous-time domain, to a discrete-time formulation.

Requirements

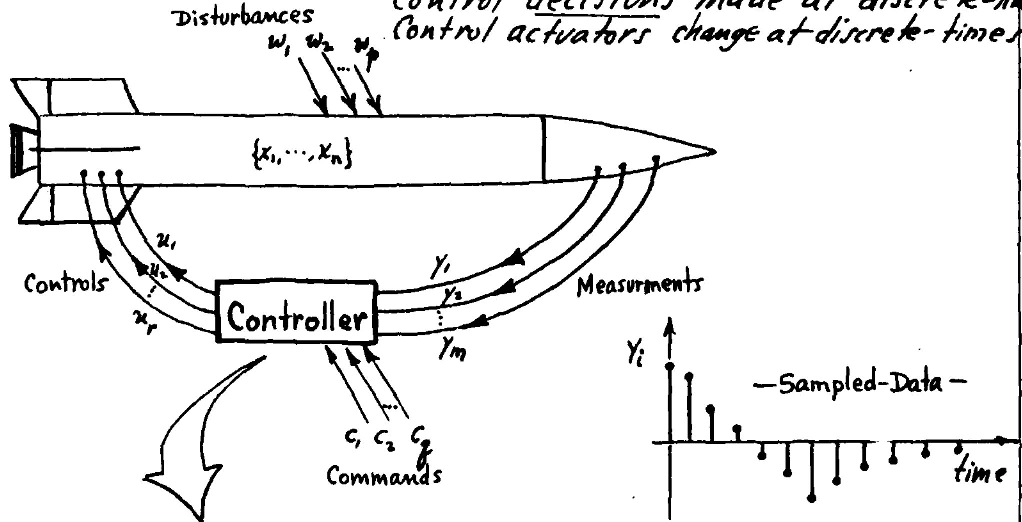
1. Waveform-mode description of disturbances
2. Discrete-time versions of disturbance state-models
3. Define class of systems and disturbances amenable to the discrete-time theory
4. Describe the state-reconstructor
5. Constraints on the structure of the discrete DAC
6. Description of the regulator and tracking control problems

<u>Note</u>: Design of DAC's is <u>not</u> required in this scope of work.

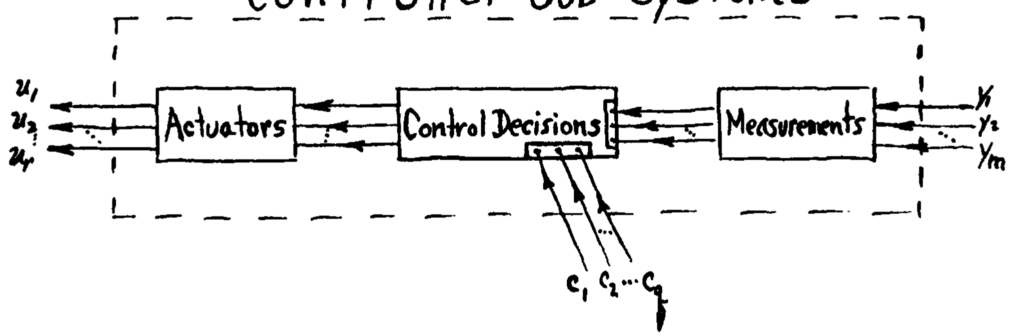
Sources of Discrete-Time Effects in Missile Guidance & Control Problems

Working Definitions:

- "Discrete-Time Effect" $\triangleq$ Measurements (includes discrete sensors) made at discrete-times, Data arrival at discrete-times, Control decisions made at discrete-times, Control actuators change at discrete-times



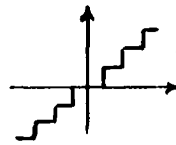
Controller Sub-Systems



- "Time-Delay (Transport-Lag) Effect" $\triangleq$ pure time-delay in transmission of information, stimuli, etc.

Sources of Discrete-Time and Time-Delay Effects in Missiles

Sensors

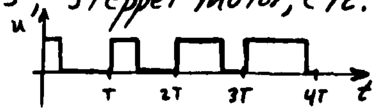
- Inherently quantized sensor data 

- Internal a/d conversion <u>in</u> the sensor
- Modulation/Demodulation in the sensor
- Digital Filtering in the sensor
- General Data Processing in the sensor (on-line correlation, etc)
- Data Reflections; doppler, pulse-radar, sector scanning
- Multiplexed Sensors (time-shared)

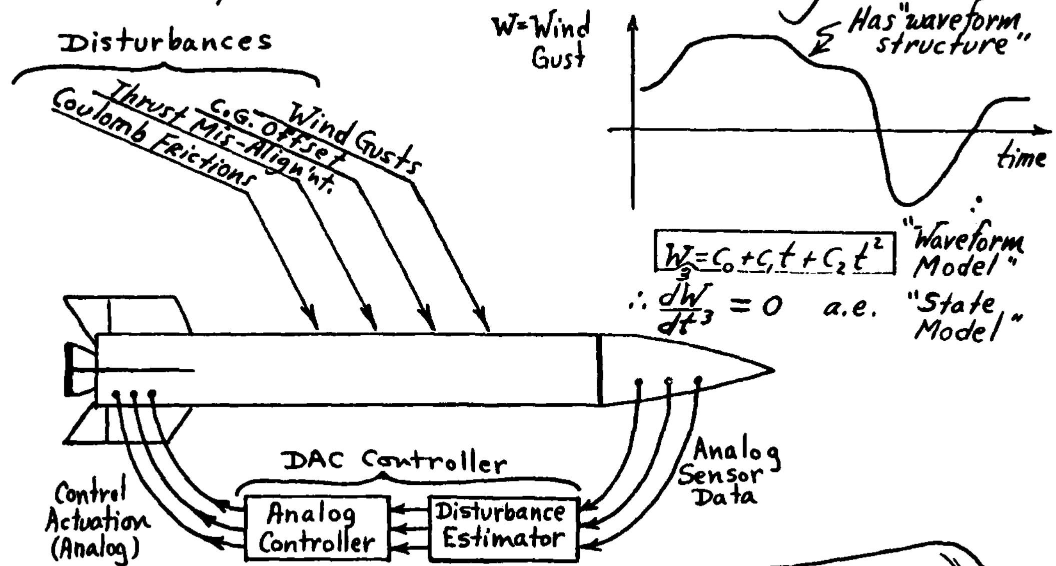
Control Decisions

- Control Digital Computer; compute times; time-shared w/other tasks
- Guidance Commands arriving at discrete-times
- Guidance Commands Multiplexed (Time-Shared) w/other missiles
- Piecewise-Constant Guidance Laws
- Control Algorithms requiring iterations; trial & error procedures; data extrapolation; parameter identification.

Control Application

- Actuator Power Source Time-Shared
- Inherently Quantized Actuator Outputs; stepper motor, etc.
- Pulse-Modulation of Actuator Signals 

Brief Review of Continuous-Time DAC Theory



Nature of DAC Controller (Disturbance-Absorption Mode)

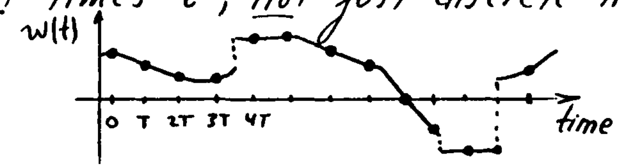
The DAC Controller generates a real-time on-line estimate of the actual (instantaneous) disturbance waveform and creates a special control action that exactly cancels-out the disturbance effect on the missile, [like classical idea of "crabbing into the wind" to cancel-out the wind-drift effect; ditto for boating example].

Remarks:

In addition to disturbance-absorption (counteraction), the DAC Controller can also be designed to minimize the effects of disturbances on the missile performance, or to make-use of the disturbances in accomplishing the missile control task.

Waveform-Mode Description of Realistic Disturbances

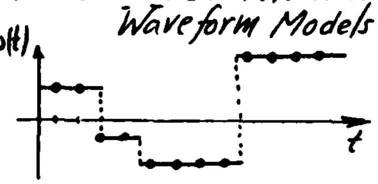
- **Remark**: The actual physical disturbances encountered by a missile are *always* "continuous-time" in nature; i.e. they exist and are well-defined for *all* times t, *not* just discrete-times $0, T, 2T, 3T, \cdots$.



The necessity of going to discrete-time models of disturbances is due to the control designers decision to employ *data-sampling* and *digital computers* in the control loop.

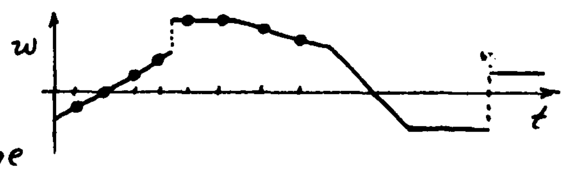
- Some Examples of Common Disturbances w/Continuous and Discrete-Time Waveform Models

1. Constant Disturbances (ie. piecewise constant)



Continuous-time: $w(t) \equiv C$

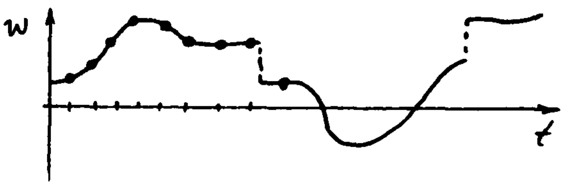
Discrete-Time: $w(nT) \equiv C \;;\; n = 0, 1, 2, \cdots$



2. Constant + Ramp Disturbances

Continuous-time: $w(t) = C_1 + C_2 t$

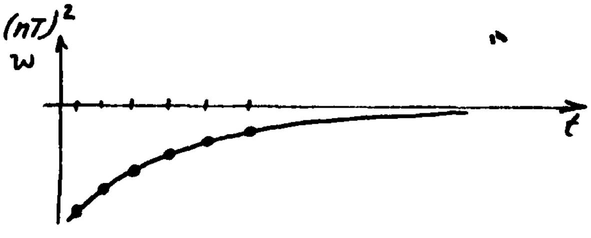
Discrete-Time: $w(nT) = C_1 + C_2 (nT)$



3. Polynomial Disturbances

Continuous-time: $w(t) = C_1 + C_2 t + C_3 t^2$

Discrete-time: $w(nT) = C_1 + C_2 (nT) + C_3 (nT)^2$



4. Exponential Disturbances

Continuous-time: $w(t) = C_1 e^{\alpha t}$

Discrete-Time: $w(nT) = C_1 e^{\alpha (nT)}$

Discrete-Time Difference Eq. Models for Disturbances

- Objective: Given the discrete-time waveform description of a disturbance, find a difference equation which that waveform satisfies (obeys).

- Some Examples

1. Constant Disturbances (piecewise constant)

Waveform description | Difference equation
$w(nT) = C$ | $\boxed{w(nT+T) - w(nT) = 0}$; a.e.

2. Constant + Ramp Disturbances

Waveform description | Difference equation
$w(nT) = c_1 + c_2(nT)$ | $\boxed{w(nT+2T) - 2\,w(nT+T) + 1\,w(nT) = 0}$; a.e.

3. Polynomial Disturbances

Waveform description | Difference equation
$w(nT) = c_1 + c_2(nT) + c_3(nT)^2$ | $\boxed{w(nT+3T) - 3\,w(nT+2T) + 3\,w(nT+T) - 1\,w(nT) = 0;}$ a.e.

- The Technique

<u>Theorem:</u> If the continuous-time disturbance function $w(t)$ obeys the differential equation $\frac{d^\rho w}{dt^\rho} + \beta_\rho \frac{d^{\rho-1} w}{dt^{\rho-1}} + \cdots + \beta_2 \frac{dw}{dt} + \beta_1 w = 0$, a.e, ($\beta_i$ = constant) then the discrete-time disturbance function $w(nT)$, $n = 0, 1, 2, \cdots$, obeys the difference equation $w(nT+\rho T) + \tilde{\beta}_\rho w(nT+(\rho-1)T) + \cdots + \tilde{\beta}_2 w(nT+T) + \tilde{\beta}_1 w(nT) = 0$, a.e, where the coefficients $\tilde{\beta}_i$, $i = 1, 2, \cdots \rho$ are calculated by the formula: $\prod_{1}^{\rho}(\tilde{\lambda} - e^{\lambda_i T}) = \tilde{\lambda}^\rho + \tilde{\beta}_\rho \tilde{\lambda}^{\rho-1} + \cdots + \tilde{\beta}_2 \tilde{\lambda} + \tilde{\beta}_1$, where the λ_i are the characteristic roots of the continuous-time characteristic equation $\lambda^\rho + \beta_\rho \lambda^{\rho-1} + \cdots + \beta_2 \lambda + \beta_1 = 0$. ∎

Discrete-Time State Models for Disturbances

- Objective: Given the difference equation model for a disturbance, find an appropriate state-variable model

- Some Examples

1. Constant Disturbances

Difference Eq. Model

$$w(nT+T) - w(nT) = 0$$

State-Variable Model

$$z_1(nT) \triangleq w(nT)$$

$$\therefore \boxed{z_1((n+1)T) = z_1(nT)}$$

2. Constant + Ramp Disturbances

Difference Eq. Model

$$w(nT+2T) - 2\,w(nT+T) + 1\,w(nT) = 0$$

State-Variable Model

$$z_1(nT) \triangleq w(nT)$$

$$z_2(nT) \triangleq w(nT+T)$$

$$\therefore \boxed{\begin{pmatrix} z_1((n+1)T) \\ z_2((n+1)T) \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} z_1(nT) \\ z_2(nT) \end{pmatrix}}$$

3. Sinusoidal Disturbances

Difference Eq. Model

$$w(nT+2T) - 2\cos\omega T\; w(nT+T) + 1\,w(nT) = 0$$

State-Variable Model

$$z_1(nT) \triangleq w(nT)$$

$$z_2(nT) \triangleq w(nT+T)$$

$$\therefore \boxed{\begin{pmatrix} z_1((n+1)T) \\ z_2((n+1)T) \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2\cos\omega T \end{bmatrix} \begin{pmatrix} z_1(nT) \\ z_2(nT) \end{pmatrix}}$$

- General Result

$$w(nT) = \tilde{H} z(nT)$$

$$z(nT+T) = \tilde{D} z(nT) + \tilde{\sigma}(nT) \;;$$

$\tilde{\sigma}(nT)$ = effect of random-like impulses which "jump" the IC of $z(\cdot)$

where

$z(nT)$ = "state" of the disturbance $w(\cdot)$ at time $t = nT$

Some Issues in Discrete-Time Disturbance Modeling

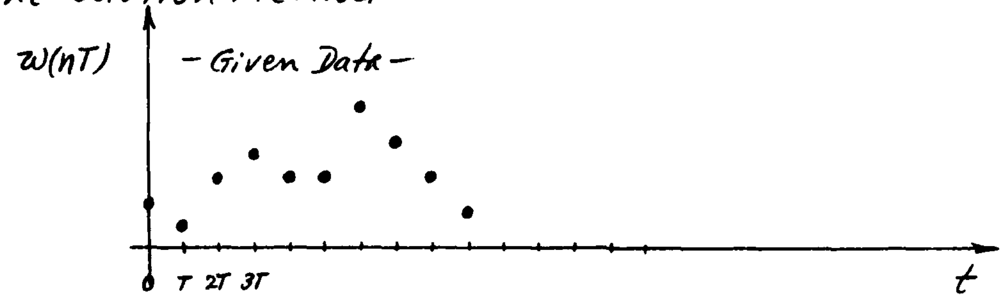
- Given a discrete-time data set $\{w(0), w(T), w(2T), w(3T), \cdots\}$, from experimental tests, find a difference equation which that data satisfies. ["answer" not unique, in general]

- Find the "most effective" difference equation for a given data set $\{w(0), w(T), w(2T), \cdots\}$. Here, effective includes considerations of "order" of the difference eq. and non-linearity (or coeff. time-variation) of the difference eq., and controller capabilities.

- Develop a scheme for on-line identification of the disturbance model for the case where the disturbance model is (slowly) changing in real-time. (Adaptive DAC)

IDENTIFICATION OF DIFFERENCE EQS. FOR DISTURBANCES FROM EXPERIMENTAL DATA $\{w(0), w(T), w(2T), \cdots, w(NT)\}$

One Solution Method



Step 1 - Assume order of the sought difference eq., starting with low value and increasing order as needed.

Example: Assume $\underbrace{w(nT+2T)}_{\text{order}=2} + \tilde{\beta}_2\, w(nT+T) + \tilde{\beta}_1\, w(nT) = 0$, ae.

Step 2 - "Fit" the given experimental data to the assumed equation in a sequental fashion, using a one-step overlap.

unknowns — given data

Example:

Case of assumed n = 2; for n = 3 each stage has 3 equations, etc.

stage 1 –
$$\begin{cases} w(2T) + \tilde{\beta}_2(3T)\, w(T) + \tilde{\beta}_1(3T)\, w(0) = 0 \\ w(3T) + \tilde{\beta}_2(3T)\, w(2T) + \tilde{\beta}_1(3T)\, w(T) = 0 \end{cases}$$

data overlap

stage 2 –
$$\begin{cases} w(3T) + \tilde{\beta}_2(4T)\, w(2T) + \tilde{\beta}_1(4T)\, w(T) = 0 \\ w(4T) + \tilde{\beta}_2(4T)\, w(3T) + \tilde{\beta}_1(4T)\, w(2T) = 0 \end{cases}$$

data overlap

stage 3 –
$$\begin{cases} w(4T) + \tilde{\beta}_2(5T)\, w(3T) + \tilde{\beta}_1(5T)\, w(2T) = 0 \\ w(5T) + \tilde{\beta}_2(5T)\, w(4T) + \tilde{\beta}_1(5T)\, w(3T) = 0 \end{cases}$$

etc.

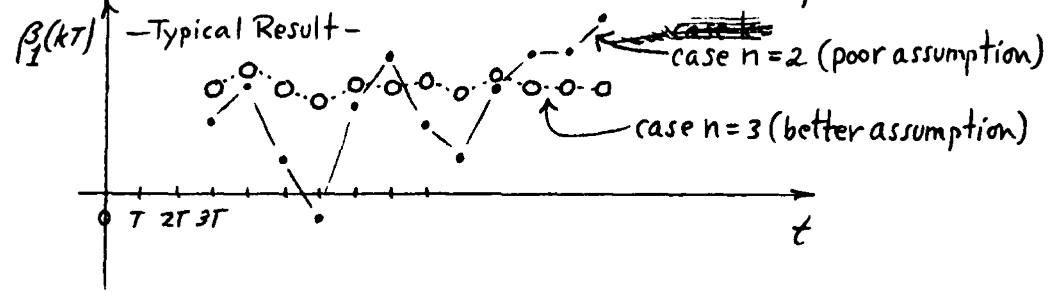
Step 3 - At each "stage", solve for the associated values of the unknown coefficients $\tilde{\beta}_i(kT)$. Example: At stage 1, solve for $(\tilde{\beta}_2(3T), \tilde{\beta}_1(3T))$, at stage 2 solve for $(\tilde{\beta}_2(4T), \tilde{\beta}_1(4T))$, etc.

Step 4 - Examine fluctuation in $\tilde{\beta}_i(kT)$ for $k = 3, 4, 5, \cdots$,

(cont'd.)

(cont'd.) If fluctuation is too high (no well-defined "steady" value) then return to Step 1 and increase guessed value of n by one.

Step 5 – Repeat above steps until the $\tilde{\beta}_i(kT)$, $(k=3,4,5\cdots)$ computed at each stage are "relatively equal" --- allowing for inevitable data noise and other imperfections.



Remark: The solution method outlined above presumes that the sought β_i are constants. If the actual β_i are time-varying the solution method must be modified. Such modifications are under study.

Class of Systems and Disturbances Being Considered in Discrete-Time DAC Theory

Shorthand notation for discrete-time equations

$$x((n+1)T) \longrightarrow E\,x(nT) \quad ; \quad E(\cdot) = \text{"shift-operator"}$$

(shifts time ahead by one sample-period T)

$$x((n+2)T) \longrightarrow E^2 x(nT)$$

etc.

- General Discrete-Time State Model for Plants

$$\begin{cases} E\,x(nT) = \tilde{A}(nT)\,x(nT) + \tilde{B}(nT)u(nT) + \tilde{F}H(nT)z(nT) + \gamma(nT) \\ y(nT) = \tilde{C}(nT)\,x(nT) + \tilde{E}(nT)u(nT) + \tilde{G}(nT)\,w(nT) \end{cases}$$

Compare with conventional continuous-time model

$$\begin{cases} \dot{x} = A(t)\,x + B(t)u(t) + F(t)H(t)z(t) \\ y = C(t)\,x + E(t)u(t) + G(t)\,w(t) \end{cases}$$

Remarks: The discrete-time model is <u>more general</u> than the continuous-time model. Moreover, the discrete-time model can be easily modified to accommodate controller/sensor "time-delays," (provided delays occur in <u>integer</u> <u>multiples</u> of the sampling period T).

- General Discrete-Time State Model for Disturbances

$$\begin{cases} w(nT) = \tilde{H}(nT)z(nT) + \tilde{L}(nT)x(nT) \\ E\,z(nT) = \tilde{D}(nT)z(nT) + \tilde{M}(nT)\,x(nT) + \tilde{\sigma}(nT) \end{cases}$$

Remarks: Same as above.
<u>Also</u>: Here "z" is the disturbance state --- <u>not</u> the z-transform variable.

These terms allow for "plant-dependent" disturbance effects

Composite Discrete-Time Model for Plant + Disturbance

Remark: The disturbance $w(t)$ is typically NOT coupled to the plant dynamics through a sample-hold arrangement. Thus, the composite model for plant + disturbance must not assume that $w(t) = w(nT)$, but rather that $w(t) = w(t)$.

- Review of Composite Model for Continuous-Time Case

$$\begin{cases} \dot{x} = Ax + Bu + Fw \\ y = Cx \end{cases}$$

$$\begin{cases} w = Hz \\ \dot{z} = Dz \end{cases}$$

$$\left(\begin{array}{c} \dot{x} \\ \hline \dot{z} \end{array}\right) = \left[\begin{array}{c|c} A & FH \\ \hline 0 & D \end{array}\right]\left(\begin{array}{c} x \\ \hline z \end{array}\right) + \left[\begin{array}{c} B \\ \hline 0 \end{array}\right]u \quad ; \quad y = [C \mid 0]\left(\begin{array}{c} x \\ \hline z \end{array}\right)$$

- Composite Model for Discrete-Time Case (Time-Invariant Case)

$$\left(\begin{array}{c} Ex \\ \hline Ez \end{array}\right) = \left[\begin{array}{c|c} e^{AT} & \int_0^T e^{A(T-\tau)} FH e^{D\tau} d\tau \\ \hline 0 & e^{DT} \end{array}\right]\left(\begin{array}{c} x(nT) \\ \hline z(nT) \end{array}\right) + \left[\begin{array}{c} \int_0^T e^{A(T-\tau)} B d\tau \\ \hline 0 \end{array}\right] u(nT)$$

$$y(nT) = [C \mid 0]\left(\begin{array}{c} x(nT) \\ \hline z(nT) \end{array}\right)$$

State-Reconstructor for Discrete-Time DAC

- Composite Plant + Disturbance Discrete-Time Model

$$\begin{pmatrix} E x \\ \hline E z \end{pmatrix} = \left[\begin{array}{c|c} \widetilde{A} & \widetilde{FH} \\ \hline O & \widetilde{D} \end{array}\right] \begin{pmatrix} x(nT) \\ \hline z(nT) \end{pmatrix} + \left[\begin{array}{c} \widetilde{B} \\ \hline O \end{array}\right] u(nT)$$

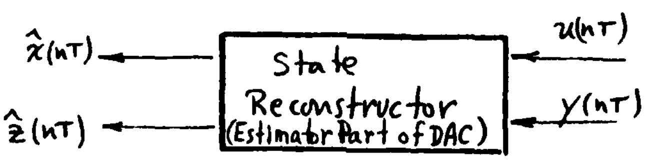
For definitions of $\widetilde{A}, \widetilde{B}, \widetilde{D}$, etc. see bottom half of page 13.

$$y(nT) = [C \mid 0] \begin{pmatrix} x(nT) \\ \hline z(nT) \end{pmatrix}$$

- Structure of a Full-Dimensional Composite Reconstructor for the Discrete-Time Case

$$\begin{pmatrix} E \hat{x} \\ \hline E \hat{z} \end{pmatrix} = \left[\begin{array}{c|c} \widetilde{A} & \widetilde{FH} \\ \hline O & \widetilde{D} \end{array}\right] \begin{pmatrix} \hat{x}(nT) \\ \hline \hat{z}(nT) \end{pmatrix} + \left[\begin{array}{c} \widetilde{B} \\ \hline O \end{array}\right] u(nT) + \left[\begin{array}{c} K_{01} \\ \hline K_{02} \end{array}\right] \left[[C \mid 0] \begin{pmatrix} \hat{x} \\ \hline \hat{z} \end{pmatrix} - \overset{\text{sensor measurement}}{y(nT)} \right]$$

$$\left.\begin{array}{l} \hat{x}(nT) \\ \hat{z}(nT) \end{array}\right\} \text{— outputs of the state reconstructor}$$



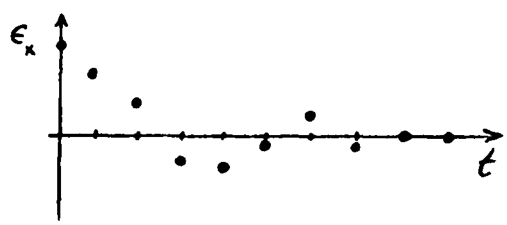
- Error Dynamics for State Reconstructor

Set: $\epsilon_x \triangleq x(nT) - \hat{x}(nT)$ ← estimation error

$\epsilon_z \triangleq z(nT) - \hat{z}(nT)$ ← " "

Then, it turns-out that

$$\begin{pmatrix} E \epsilon_x \\ \hline E \epsilon_z \end{pmatrix} = \left[\begin{array}{c|c} \widetilde{A} + K_{01} C & \widetilde{FH} \\ \hline K_{02} C & \widetilde{D} \end{array}\right] \begin{pmatrix} \epsilon_x(nT) \\ \hline \epsilon_z(nT) \end{pmatrix};$$



<u>Design problem</u>: Choose K_{01}, K_{02} to make $\epsilon_x \to 0$; $\epsilon_z \to 0$."

<u>Design procedure</u>: In time-invariant case, use standard pole-assignment techniques to place all eigenvalues of $\left[\begin{array}{c|c} \widetilde{A} + K_{01} C & \widetilde{FH} \\ \hline K_{02} C & \widetilde{D} \end{array}\right]$ <u>inside</u>* the unit circle in the complex plane. The <u>"Unified Canonical Form"</u> is directly applicable.

* $\lambda_i = 0$ yields "deadbeat response" --- an attractive behavior for $\epsilon(nT)$

A Reduced-Order State-Reconstructor for Discrete-Time DAC

- Plant Model (Same as before)

- A Special Coordinate Transformation

$$\left(\frac{x}{z}\right) = \left[\begin{array}{c|c} T_{12} & C^{\#T} - T_{12}\Sigma \\ \hline T_{22} & -T_{22}\Sigma \end{array}\right]\left(\frac{\xi}{y}\right) \quad ; \quad \xi = \text{a "new" vector-variable of dimension "} n+p-m \text{"}$$

Σ = arbitrary

where $T_{12} = n \times (n+p-m)$, $T_{22} = p \times (n+p-m)$ $\Big\}$ such that

$$\begin{cases} [C \mid 0]\left[\frac{T_{12}}{T_{22}}\right] = 0 \\ \operatorname{rank}\left[\frac{T_{12}}{T_{22}}\right] = n+p-m \end{cases}$$

The inverse transformation

$$\left(\frac{\xi}{y}\right) = \left[\begin{array}{c|c} \Sigma C + \bar{T}_{12} & \bar{T}_{22} \\ \hline C & 0 \end{array}\right]\left(\frac{x}{z}\right) \; ; \quad \begin{aligned} \bar{T}_{12} &= (T_{12}^T T_{12} + T_{22}^T T_{22})^{-1} T_{12}^T \\ \bar{T}_{22} &= (T_{12}^T T_{12} + T_{22}^T T_{22})^{-1} T_{22}^T \end{aligned}$$

- Transformed Plant Dynamics

$$\left(\frac{E\xi}{Ey}\right) = \left[\begin{array}{c|c} (\Sigma C + \bar{T}_{12})(\tilde{A}T_{12} + \tilde{F}HT_{22}) + \bar{T}_{22}\tilde{D}T_{22} & (\Sigma C + \bar{T}_{12})[\tilde{A}(C^{\#T} - T_{12}\Sigma) - \tilde{F}HT_{22}\Sigma] - \bar{T}_{22}\tilde{D}T_{22}\Sigma \\ \hline C(\tilde{A}T_{12} + \tilde{F}HT_{22}) & C[\tilde{A}(C^{\#T} - T_{12}\Sigma) - \tilde{F}HT_{22}\Sigma \end{array}\right]\left(\frac{\xi(nT)}{y(nT)}\right) + \left[\frac{(\Sigma C + \bar{T}_{12})\tilde{B}}{C\tilde{B}}\right] u(nT)$$

- Final Recipe for the State-Reconstructor

"Assembly" Eqs.

$$\begin{cases} \hat{x}(nT) = T_{12}\,\xi(nT) + [C^{\#T} - T_{12}\Sigma]\,y(nT) \\ \hat{z}(nT) = T_{22}\,\xi(nT) - T_{22}\Sigma\,y(nT) \end{cases}$$

$(\Sigma C + \bar{T}_{12})\hat{A}\,C^{\#T}$

$-(\Sigma C + \bar{T}_{12})[\tilde{A}(T_{12}\Sigma) + \hat{F}HT_{12}\Sigma]$

$-(\Sigma C + \bar{T}_{12})(\tilde{A}T_{12} + \hat{F}HT_{22})\Sigma$

"Filter" Eq. ((n+p-m)-order difference eq.)

$$E\,\xi(nT) = (\tilde{\mathcal{D}} + \Sigma\tilde{\mathcal{H}})\,\xi(nT) + \left[(\bar{T}_{12} + \Sigma C)(\tilde{A}C^{\#T}) - (\tilde{\mathcal{D}} + \Sigma\tilde{\mathcal{H}})\Sigma\right] y(nT) + (\bar{T}_{12} + \Sigma C)\tilde{B}\,u(nT)$$

where

$$\tilde{\mathcal{D}} = \bar{T}_{12}(\tilde{A}T_{12} + \tilde{F}HT_{12}) + \bar{T}_{22}\tilde{D}T_{22}$$

$$\tilde{\mathcal{H}} = C(\tilde{A}T_{12} + \tilde{F}HT_{22}) \; .$$

- Error Dynamics

$$E\epsilon(nT) = (\tilde{\mathcal{D}} + \Sigma\tilde{\mathcal{H}})\,\epsilon(nT) \; ; \text{ a.e.;} \; \begin{aligned} (x - \hat{x}) &= T_{12}\,\epsilon(nT) \\ (z - \hat{z}) &= T_{22}\,\epsilon(nT) \end{aligned} \; .$$

Constraints on the Structure of the Discrete DAC

The application of Disturbance-Accommodating Control Theory to systems with data-sampling and digital data-processing opens up a wider field of candidate implementation schemes. In particular, the digital data-processing unit (i.e. microprocessor, microcomputer, etc.) contains memory capabilities which enable the control law to "remember" past values of plant outputs, disturbance behavior, etc. This memory capability can be profitably used to enhance the controllers adaptability qualities thereby allowing, for instance, the controller to "learn" about a changing disturbance environment and automatically adapt to it.

As a consequence of the facts cited above, the structure of the discrete DAC controller can be represented in the following general form:

$$\begin{aligned} u(nT) = f\Big(& y(nT), y((n-1)T), \cdots, y((n-k)T);\ \hat{x}(nT), \hat{x}((n-1)T), \cdots, \hat{x}((n-k)T);\ \hat{z}(nT), \hat{z}((n-1)T), \cdots, \hat{z}((n-k)T); \\ & ;\ y_c(nT), y_c((n-1)T), \cdots, y_c((n-k)T);\ \hat{c}(nT), \hat{c}((n-1)T), \cdots, \hat{c}((n-k)T); \\ & ;\ w_m(nT), w_m((n-1)T), \cdots, w_m((n-k)T);\ \hat{z}_m(nT), \hat{z}_m((n-1)T), \cdots, \hat{z}_m((n-k)T);\ nT \Big) \end{aligned}$$

where $y_c(\cdot)$ denotes set-point or servo-commands, $\hat{c}(\cdot)$ denotes the "state" of the set-point or servo-command dynamical process, $w_m(\cdot)$ denotes directly measurable disturbances (if there are any), and $\hat{z}_m(\cdot)$ denotes the "state" of $w_m(\cdot)$. Note that this controller expression contains data extending kT units (sample-periods) <u>into</u> <u>the</u> <u>past</u>.

Description of Regulator and Servo-Tracking Control Problems for Discrete-Time DAC Applications

- Plant + Disturbance Model for Discrete-Time

$$E\,x(nT) = \tilde{A}\,x(nT) + \tilde{B}u(nT) + \widetilde{FH}\,z(nT) + \gamma(nT)$$

$$E\,z(nT) = \tilde{D}z(nT) + \tilde{\sigma}(nT)$$

$$y(nT) = \tilde{C}\,x(nT)$$

- Set-Point Regulator Control Problems

Given: Set of admissible set-points

$$X_{sp} = \{x \mid x = x_{sp} \text{ is admissible set-point}\}$$

$$Y_{sp} = \{y \mid y = y_{sp} \text{ is admissible set-point}\}$$

Control Task: Design $u(nT)$ such that $x(nT) \to x_{sp}$, $n \to \infty$, $\forall$ admissible $z(nT)$ and $\forall\, x(0)$, for each given $x_{sp} \in X_{sp}$ [or $y_{sp} \in Y_{sp}$].

Remark – The <u>quality</u> of the motion $x(nT) \to x_{sp}$ may also be specified.

- Servo-Tracking Control Problems

Given: Set of expected servo-commands $\{y_c(nT)\}$ described by:

$$\begin{cases} y_c(nT) = S\,c(nT) \\ E\,c(nT) = \tilde{R}\,c(nT) + \tilde{\mu}(nT)\,; \quad \tilde{R} = e^{RT}. \end{cases}$$

Control Task: Design $u(nT)$ such that $y(nT) \to y_c(nT)$ "promptly", $\forall$ admissible $y_c(nT)$, $\forall$ admissible $z(nT)$, and $\forall\, x(0)$.

- Modes of Disturbance-Accommodation
  - (i) Disturbance Absorption (Cancellation)
  - (ii) Disturbance Minimization
  - (iii) Disturbance Utilization
  - (iv) Multi-Mode Accommodation

Conclusions

The DAC theory developed in this study provides a sound mathematical basis upon which one can proceed to derive engineering design rules for synthesizing Disturbance-Accommodating Controllers for digital-controlled missiles. The next step in this development program would be the detailed derivation of step-by-step algorithms for designing DAC controllers to accomplish disturbance absorption and disturbance minimization in sampled-data, digitally-controlled missile applications.